Optimization and Design of Geodetic Networks

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I. Estimability Analyses of the Free Networks of Differential Range Observations to GPS Satellites

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1. INTRODUCTION

Historically in geodesy, the conventional analysis of satellite networks has been carried out in three distinct steps. First, satellite orbits are computed from observations at some ground tracking stations whose coordinates are precisely known. Second, these computed orbits are taken for granted or slightly relaxed and used for the computation of the coordinates of other ground stations which have carried out additional observations to the same satellites. Third, this process is repeated as more observations become available.

The use of satellite observations for the establishment of satellite networks is intimately connected with the concepts of reference frames. Hence in the foregoing sequential process, solutions where adequate precautions with respect to the estimable parameters in the process are not exercised would yield values which may not necessarily refer to the reference frame of interest. A major cause for this is the rank deficiency encountered in the normal equations matrix formed without appropriate constraints closely related to the mathematical models used to relate the observables with the unknowns for the geodetic problem in hand, i.e. the computation of ground and/or satellite coordinates. For instance, it is well known that due to the datum defect satellite observations such as ranges or Doppler observations do not contain any information about the translational or rotational degrees of freedom of the satellite network. In addition due to the configuration defect notion introduced e.g. by [Pelzer 1974], satellite observation equations are deficient with respect to the network configuration if a sufficient number of simultaneous observations from a mininum number of ground stations is not fulfilled.

The question of estimable and invariant quantities in satellite networks is not new. Veis (1960) was first to raise the question of estimability and the existence of singularities in satellite networks. However, it was Rinner (1966) and Meissl (1969) who revived in Europe the geodetic interest in estimability and some time later in the United States Blaha (1971a, 1971b) with his studies on the critical configurations for fundamental satellite range networks.

The main idea expressed therein was to have the mathematical models set up as generally as possible and subsequently investigate which additional information in the form of inner and minimal constraints was needed in order to make the original rank deficient normal equations invertable. By contrast, Aardoom (1970 and 1971) in his studies, also applied to satellite ranging, favoured having the mathematical model directly expressed in the maximum number of estimable parameters, thus circumventing the need to find proper inner constraints (which is not always an easy task). Up to recently questions concerning estimability in satellite geodetic networks studied in the dynamic mode were either ignored completely, stated only explicitly or more often tacitly assumed to be known. Some attempts e.g. Brown and Trotter (1969) and Arur (1977) did not pursue the subject to its fullest extent and therefore did not fully clarify the problem. The latest systematic attempt to handle this problem was presented by Grafarend and Livieratos (1978), Grafarend and Heinz (1978) and Van Gelder (1978), with numerical tests presented e.g. by Grafarend et al., (1979), Grafarend et al., (1982), and Grafarend et al., (1983).

It is in view of this general background that we have attempted here to investigate the different rank deficiencies of external type in satellite networks based on differential GPS observations. Rank defect situations are reviewed here for the geometric mode only. Also the addition of instrumental unknowns and their effect with respect to rank deficiencies in the differential GPS observational equations has also been analyzed by considering corrections for frequency instability, clock errors and cycle ambiguity parameters.

2. TYPES OF RANK DEFICIENCIES

Let us begin with some definitions of rank deficiencies that we shall be dealing with in the sequel. Following Grafarend and Livieratos (1978), let

$$x \rightarrow \ell + v = Ax \tag{2.1}$$

be a linear mapping of a non-stochastic unknown vector $x \epsilon X$ into a stochastic vector of observations $\ell \epsilon L$ where

$$X \equiv R^m \tag{2.2a}$$

$$L \equiv R^n \tag{2.2b}$$

are spaces over the real numbers of dimensions m and n respectively, A is the non-stochastic configuration (design) matrix of the network and v is the residual vector.

Denoting the rank of A by r(A), which always satisfies the conditions

$$r(A) \leq m \leq n \tag{2.3}$$

rank deficiency with respect to surjectivity ("onto" mapping) is defined as

$$d_s(A) = n - r(A) \tag{2.4a}$$

whereas **rank deficiency with respect to injectivity** ("one-to-one" mapping) is defined as

$$d_i(A) = m - r(A). \tag{2.4b}$$

In simple terms, if there are more observations than unknowns then the system of equations (2.1) is rank deficient with respect to surjectivity in which case the minimization of the least squares norm, i.e.

$$||\ell - A\hat{x}|| = \min$$

removes the surjectivity defect.

However, what is of interest here are rank defects with respect to injectivity which destroy, as already mentioned, the uniqueness of the inversion of the normal equations matrix of the satellite observational equations. An indication of injectivity deficiency is the zero determinant of the normal equations matrix which cannot be inverted by the Caley inverse (Bjerhammar 1973; Vaníček and Krakiwsky 1982).

For the analysis of satellite observations, there is a common distinction of three types of injectivity defects;

Datum Defects (d-defects)

D-defects are closely connected with the origin of the reference system which is undefined through the observations. The d-defect is inherent in any geodetic observation equation whereby absolute coordinates are implemented as unknowns. For example, from the terrestrial triangulation networks, it is well known that direction observations are invariant with respect to translation and scale changes in which case coordinates cannot be computed from these observations alone. A similar example from satellite networks are range or Doppler observations which are invariant with respect to the coordinate system, i.e. they do not provide any information about its origin or its orientation.

To overcome the situation of d-defects additional information about the coordinate system lacking in the observations is needed in the form of constraints on the parameters, with the choice and the nature of the constraints enforced generally having a direct influence in the coordinate system in which the solution is obtained.

Configuration Defects (c-defects)

C-defects often arise when the number or the choice of the observations are not sufficient in order to determine uniquely the size and shape of the network. In the example of a triangulation network for instance besides the foregoing d-defect, there is a c-defect arising from missing information about scale, in which case the c-defect can be removed by measuring one distance. With satellite networks, up to a few years ago within the conventional approach c-defects had not become apparent since geodesists assumed a model satellite orbit as the real one. However, with the extensive analyses of satellite Doppler networks in recent years, it is now known that only a sufficient number of **simultaneous observations** can remove the c-defects.

Ill-Conditioning Defects (i-defects)

In certain cases of satellite networks when ground and/or satellite points are situated on special configurations, a unique adjustment is impossible even if the number of observations is sufficient. Such critical configurations, result in singular solutions, i.e. the basic system of normal equations is ill-conditioned.

Blaha (1971b) and Tsimis (1973) have studied such critical configurations for range satellite observations from the analytical point of view in an attempt to detect the singularity for theoretical cases and establish rules to avoid it. However in more practical terms what is of interest, is the detection of the i-defect when the normal equations matrix for a system of observational equations is to be numerically inverted. Grafarend et al., (1982) have indicated that a direct measure of i-defect is the spectral number

$$s = \frac{\lambda_{min}}{\lambda_{max}} - 1$$

where λ_{min}, λ_{max} are respectively the minimum and the maximum eigenvalues of the normal equation matrix. If s is zero then the width of the spectrum of the normal equations matrix is zero and the matrix can be inverted optimally.

3. RANK DEFICIENCIES OF FREE NETWORKS BASED ON DIFFERENTIAL RANGE GPS OBSERVATIONS

In the investigation of rank deficiencies of free networks based on differential range observations to GPS satellites in the geometric mode the various unknown parameters can be grouped in three categories:

a) station coordinate parameters
b) satellite coordinate parameters
c) instrumental type parameters like clock errors, and other non-geometric parameters like ionospheric and tropospheric errors.

The parameters in (b) above usually are cartesian coordinates expressed in the same reference frame the station coordinates are expressed in. By contrast, in the semi-dynamic mode, the cartesian coordinates in (b) are replaced by six Keplerian elements. In addition in the dynamic mode analyses, a group of Earth parameters such as the Earth's rotation rate, the Earth's gravitational constant, the J_2 term and the GAST at the reference epoch of the Keplerian elements is added to the list of unknown parameters. Further parameters like polar motion, nutation, and precession and Earth rotation parameters can also be included leading to a more unified model but with expense of added complexity.

3.1 Determination of Station and Satellite Coordinates

The basic measurable quantity of GPS point positioning is a function of the magnitude of the instantaneous range vector between a ground station and a satellite. In differential positioning, suitable differences of range vector magnitudes form the observables. Although a receiving system may not be instrumented to perform direct differencing of ranges, its operation may be mathematically described as such. Actually, because of timing errors and delays in the satellite and ground station equipment and the effects of signal propagation through the ionosphere and troposphere, the measured signal is a "pseudorange". In the sequel, we shall neglect temporarily all timing and refraction errors in the following analyses and hence refer to the observations simply as ranges.

Here we shall seek to formulate the linear mathematical model that relates the position vector of each observing station to differential ranges (obtained either by differencing the ranges obtained via the L_1/L_2 timing--using either the code timing [Spilker, 1978] or the reconstructed carrier timing [Bossler et al., 1980]--or directly by the interferometric technique [Counselman et al., 1982]). The mathematical model for multistation differential range observations to GPS satellites has been discussed in Langley et al., (1984).

Starting from the mathematical model for ranging from a single point P_α ,

$$\rho^i_\alpha = \vec{e}^{\,i}_\alpha \vec{r}^{\,i} - \vec{e}^{\,i}_\alpha \vec{R}_\alpha \quad , \tag{3.1}$$

where the subscript indicates the participating station and the superscript the participating satellite position S^i, the observation equation for a differential range measurement from a pair of ground stations P_α and P_β reads

$$\begin{aligned} \Delta\rho^i_{\alpha\beta} &= (\vec{e}^{\,i}_\beta - \vec{e}^{\,i}_\alpha)\vec{r}^{\,i} + \vec{e}^{\,i}_\alpha \vec{R}_\alpha - \vec{e}^{\,i}_\beta \vec{R}_\beta \\ &= \Delta\vec{u}^{\,i}_{\alpha\beta}\vec{r}^{\,i} + \vec{e}^{\,i}_\alpha R_\alpha - \vec{e}^{\,i}_\beta \vec{R}_\beta \end{aligned} \tag{3.2}$$

where $\vec{e}^{\,i}_\alpha$, $\vec{e}^{\,i}_\beta$ are unit vectors from the ground stations to the satellite position; $\vec{R}_\alpha$, $\vec{R}_\beta$ are the station position vectors

$$\vec{R}_\alpha = [x_\alpha, y_\alpha, z_\alpha]^T \tag{3.3a}$$

$$\vec{R}_\beta = [x_\beta, y_\beta, z_\beta]^T \tag{3.3b}$$

given in the earth-fixed coordinate system; $\vec{r}^{\,i}$ is the satellite position vector

$$\vec{r}^{\,i} = [X^i, Y^i, Z^i]^T \tag{3.4}$$

given also in the earth-fixed coordinate system.

In a free network adjustment both station and satellite coordinates are considered unknowns, i.e. the sought parameters from equation (3.2) are

x_g, y_g, z_g : for every ground point

X^s, Y^s, Z^s : for every satellite point.

However, like range observations, these observations are invariant with respect to the origin and the orientation of the coordinate system a fact that leads to a d-defect of six (three translation and three rotation parameters).

In order to avoid configuration defects in a GPS satellite network of differential range observations, these simultaneous observations have to fulfill certain inequalities which are reviewed herein. There are three types of situations to be examined with respect ot c-defects. That is considering

i) ground stations unknown, satellite points known;
ii) ground stations known, satellite points unknown;
iii) both ground stations and satellite points unknown (free network)

In each case, the number of ground stations is denoted by g. Each station is assumed to observe s satellite points on different GPS satellites (different passes) and/or different times. Since we are dealing with differential range observations, simultaneous observations from g ground stations lead to $n=s(g-1)$ independent differential range observations. This is easy to see since a network of g stations has $g!/[2!(g-2)!]$ baselines which, if they all co-observe simultaneously, can reproduce (by cross-correlation of all station data at all possible pairs) the same number of differential range observations. However these observations are not functionally independent; for each network triangle of stations 1, 2 and 3 observing the same satellite point i, a condition of the form

$$\Delta\rho^{i}_{12} + \Delta\rho^{i}_{23} + \Delta\rho^{i}_{31} = 0$$

holds. That is, an independent complete set of observations can be provided e.g. by the set

$$\Delta\rho^{i}_{1\ell} = \rho^{i}_{\ell} - \rho^{i}_{1} \, , \quad \ell = 2, 3, \ldots, g$$

since any other differential range can be expressed in terms of this set, e.g.

$$\Delta\rho^{i}_{k\ell} = \rho^{i}_{\ell} - \rho^{i}_{k} = (\rho^{i}_{\ell} - \rho^{i}_{1}) - (\rho^{i}_{k} - \rho^{i}_{1}) = \Delta\rho^{i}_{1\ell} - \Delta\rho^{i}_{1k} \, .$$

Therefore the total number of independent differential range observations for s observational epochs is $n = s(g-1)$. Then for the three situations above, we end up with the following estimability models.

Model G1: Ground unknown, satellite points known: In this case, we assume the use of a precise or broadcast ephemeris which is taken as granted for the computations of satellite coordinates. In this situation, a minimum of three known satellite positions of a satellite or its "known" orbit define completely the reference frame as geocentric, with axes oriented along the Earth's rotation axis and in the direction of the Mean Greenwich meridian and the scale resulting from Kepler's third law and the GM constant. This leads to $m=3g$ as the number of estimable unknown parameters for observation equations of the form (3.2). The adjustment postulate $n \geq m$ then leads to the fundamental inequality:

$$s(g-1) \geq 3g \qquad (g > 1) \tag{3.5}$$

which characterizes the set of configuration constraints, in the satellite networks of differential range GPS observations of type G1. A set of solutions for (3.5) is

$g = 2 \quad , \quad s \geq 6$
$g = 3 \quad , \quad s \geq 5$
$g \quad 4 \quad , \quad s \geq 4$
$\ldots \qquad\qquad \ldots$
$g \rightarrow \infty \quad , \quad s \geq 3$

in which cases the c-defect is removed.

Model G2: Ground stations known, satellite points unknown: The condition for zero c-defect in this case is

$$s(g-1) \geq 3s \tag{3.6}$$

or

$$s(g-4) \geq 0$$

which with $g > 4$ always holds irrespective of the number of the observed satellite points.

Model G3: Ground stations and satellite points unknown: In this case, we have to take into account the d-defect of six resulting from the inability of the observations to provide information about the translational and rotational degrees of freedom of the satellite network. That is, the shape and size of a rigid network of g stations is sufficiently described by 3g-6 parameters. This can easily be seen for instance from the simple case of a three-station network whose shape and size is fully described by three parameters, e.g. the lengths of its three sides. In order to attach a new station to this network, at least three new parameters are needed, e.g. three distances to three "old" stations. Therefore, the total number of parameters for g stations is $3+3\cdot(g-3)=3g-6$. Alternatively if a reference frame is used there exist 3g coordinates but the six degrees of freedom for frame rotation and translation have to be subtracted. That is, we have to subtract the d-defect from the number of estimable parameters in order to investigate the c-defects.

The fundamental inequality in this case has the form

$$s(g-1) \geq 3g + 3s - 6 \tag{3.7}$$

which has no solution for $g \leq 4$. For $g > 4$ a set of solutions is

$$\begin{array}{lcl} g = 5 & , & s \geq 9 \\ g = 6 & , & s \geq 6 \\ \dots & & \dots \\ g \rightarrow \infty & , & s \geq 3 \end{array}$$

which give the number of ground stations and satellite points required in order to avoid c-defects in the G3 type model.

3.2 Determinations of Station, Satellite and Non-Geometric Parameters

The previous fundamental inequalities can be extended in a second setup of estimability analysis whereby additional non-geometric parameters are introduced in the observation equation (3.2). The most common non-geometrical parameters are instrumental and atmospheric type parameters. In the following development, we have assumed that atmospheric errors have been accounted for in some intermediate stage and therefore we shall restrict ourselves to adding in equation (3.2) the contribution of only instrumental type parameters like satellite and receiver clock errors. The parameterization of clock errors may vary significantly for different oscillator standards used for the GPS satellite clocks and the various receiver equipment systems. In the following, we have assumed that both satellite and receiver clock errors may be represented by an algebraic polynomial in time as (c.f. Davidson et al., 1983; and Langley et al., 1984).

$$\Delta t^s = a_o + a_1(\tau-\tau_o) + a_2(\tau-\tau_o)^2 \quad (3.8a)$$

$$\Delta T_g = A_o + A_1(\tau-\tau_o) + A_2(\tau-\tau_o)^2 \quad (3.8b)$$

where τ denotes the conventional "GPS time" and τ_o is some reference epoch for the particular set of coefficients. It should be noted that here we have chosen to work with the "conventional GPS time" scale rather than the receiver time scales where Langley et al., (1984) have based the polynomial representation (3.8) because it makes clearer which of these parameters are actually estimable when they are included into the model. In this case three clock error parameters for each station introduce 3(g-1) clock parameters for the network of g stations. This can be easily seen for instance for the simple case whereby the station clocks lack synchronization, i.e. only differences of clock offsets appear in the observation equations. In this case, if A_{io} is the clock offset at station i then the set $A_{i1,o}=A_{i,o}-A_{1,o}$ $i=2,3,\ldots,g$ is a complete independent set of relative offsets in the sense that any other relative offsets can be expressed in terms of this set, e.g.

$$A_{i,o} - A_{j,o} = A_{i,o} - A_{1,o} - A_{j,o} + A_{1,o} = A_{i1,o} - A_{j1,o}$$

that is, clock offsets introduce g-1 parameters for a network of g stations.

Langley et al., (1984) have extended equation (3.2) to include clock errors in the form of equation (3.8) leading to a hybrid observational equation of the form

$$\begin{aligned}\Delta\rho^i_{\alpha\beta} &= \vec{e}^{\,i}_{\beta}\vec{r}^{\,i} - \vec{e}^{\,i}_{\alpha}\vec{r}^{\,i} - \vec{e}^{\,i}_{\beta}\vec{R}_2 + \vec{e}^{\,i}_{\alpha}\vec{R}_1 \\ &- c[A_{\alpha o} + A_{\alpha 1}(\tau-\tau_o) + A_{\alpha 2}(\tau-\tau_o)^2] \\ &+ c[A_{\beta o} + A_{\beta 1}(\tau-\tau_o) + A_{\beta 2}(\tau-\tau_o)^2] \\ &= \Delta\vec{u}^{\,i}_{\alpha\beta}\vec{r}^{\,i} + \vec{e}^{\,i}_{\alpha}\vec{R}_1 - \vec{e}^{\,i}_{\beta}\vec{R}_2 - \vec{B}_{\alpha}\vec{Q}_{\alpha} + \vec{B}_{\beta}\vec{Q}_{\beta} \quad (3.9)\end{aligned}$$

where c is the speed of light, and

$$\vec{B}_g = [b_{g1}\ b_{g2}\ b_{g3}]\ ,\ g = \alpha,\beta \quad (3.10)$$

with

$$\begin{aligned} b_{g1} &= c \\ b_{g2} &= c[\tau - \tau_o] \quad (3.11) \\ b_{g3} &= c[\tau - \tau_o]^2 \end{aligned}$$

and

$$\vec{Q}_g = [A_{g0}\ A_{g1}\ A_{g2}]^T \quad (3.12)$$

is the vector of unknown clock coefficients.

Note that satellite clock error parameters are not estimable from differential range observations.

In view of the additional parameters (3.12), the models for the hybrid case corresponding to the simplified models G1 to G3 take the following form.

Model HG1: Model G1 with three clock error parameters for each ground station: In this case, the number of estimable unknown parameters increase by 3(g-1) leading to the fundamental inequality

$$s(g-1) \geq 6g - 3 \quad (g > 2) \quad . \tag{3.13}$$

A set of solutions for (3.13) is

$$\begin{array}{lcl} g = 2 & , & s \geq 9 \\ g = 3 & , & s \geq 8 \\ \dots & & \dots \\ g \rightarrow \infty & , & s \geq 6 \end{array}$$

Model HG2: Model G2 with three clock error parameters for each ground station: The fundamental inequality for this case reads

$$s(g-1) \geq 3g + 3s - 3 \tag{3.13}$$

or

$$s(g-4) \geq 3(g-1)$$

which has no solution for $g \leq 4$. For $g > 4$ a set of solutions for (3.14) is

$$\begin{array}{lcl} g = 5 & , & s \geq 12 \\ g = 6 & , & s \geq 8 \\ g = 7 & , & s \geq 6 \\ \dots & & \dots \\ g \rightarrow \infty & , & s \geq 3 \end{array}$$

Model HG3: Model G3 with three clock error parameters for each ground station: The condition for zero c-defect in this case is

$$s(g-1) \geq 6g + 3s - 9 \tag{3.15}$$

or

$$s(g-4) \geq 6g - 9$$

which again has no solution for $g \leq 4$. For $g > 4$ a set of solutions is

$$\begin{array}{lcl} g = 5 & , & s \geq 21 \\ g = 6 & , & s \geq 14 \\ \dots & & \dots \\ g \rightarrow \infty & , & s \geq 6 \end{array}$$

Model HG4: Model HG3 with one ambiguity parameter per satellite per observing session: This is a typical case which may arise when GPS differential range measurements are obtained with GPS receiver systems like the Macrometer V-1000 [Goad and Ramondi, 1983]. Due to the nature of the Macrometer observable, one has to introduce one ambiguity parameter for each satellite and for each observing session (c.f. Langley et al., 1984).

TABLE 3.1. Review of Configuration Defect Conditions for GPS Differential Range Observations

$s(g-1) \geq 3g + C_g(g-1) + C_N \cdot p$	... satellite coordinates known
$s(g-1) \geq C_g(g-1) + 3 \cdot s + C_N \cdot p$	... ground coordinates known
$s(g-1) \geq 3g + C_g(g-1) + 3 \cdot s + C_N \cdot p - 6$	... both satellite and ground coordinates unknown

g - number of ground stations
s - number of satellite points
C_g - number of clock parameters per station
p - number of satellites (satellite passes) per session
C_N - number of ambiguity parameters per satellite and observing session (e.g. Macrometer V-1000, $C_N = 1$)

If p is the number of satellites observed within the same observing session and s is again the number of satellite points being observed,then the general condition to avoid c-defects is

$$s(g-1) \geq 6g + 3s + p - 9 \tag{3.16}$$

or

$$s(g-4) \geq 6g + p - 9$$

which for each p gives a set of relations between the required number of ground stations and satellite points. For instance,

for $p = 1 \rightarrow s(g-4) \geq 6g - 8$

$$\begin{array}{ll} g = 5 \quad , & s \geq 22 \\ g = 6 \quad , & s \geq 14 \\ \ldots & \ldots \\ g \rightarrow \infty \quad , & s \geq 6 \end{array}$$

for $p = 2 \rightarrow s(g-4) \geq 6g - 7$

$$\begin{array}{ll} g = 5 \quad , & s \geq 23 \\ g = 6 \quad , & s \geq 15 \\ \ldots & \ldots \\ g \rightarrow \infty. \quad , & s \geq 6 \end{array}$$

The fundamental inequalities reviewed thus far are summarized in Table 3.1.

4. ESTIMABILITY ANALYSIS

4.1 PATTERNS OF OBSERVATIONS FOR MOVING STATIONS

Consider a network of simultaneously co-observing stations, consisting of two subnetworks where each subnetwork co-observes a set of different satellites. Utilizing the results of the previous section, a sufficient number of observation epochs leads to the estimation of the shape and size of each subnetwork independently plus the instantaneous positions of the satellites with respect to the subnetwork observing them. Obviously if the relative position of the two subnetworks were known, then at each epoch the relative position of the satellites could be interrelated for the estimation of their relative configuration. Thus in order to find the relative position of the two subnetworks, additional observations are needed with subdivisions of the network to new subnetworks.

In order to define the concepts of an observational design in such a case, the following terms will be used in the sequel. The total number (set) of observations where stations co-observe according to the same subdivision of subnetworks is said to constitute a session. Two subnetworks from different sessions are said to be directly connected if they have at least three stations in common. Two subnetworks of a network are indirectly connected if they belong to a sequence of successive directly connected subnetworks. If all the subnetworks of a network are indirecly connected in an observational design, then the shape and size of the network is fully determined. This concept of network design can be illustrated with the following example.

Consider the case of a network of twelve stations co-observing a total of six GPS satellites at 18 observational epochs. The observing campaign is divided into two sessions of 9 epochs each. For each of the two sessions, the network is subdivided into two subnetworks, with the stations of each subnetwork co-observing the same satellite at each epoch. The subdivision of the network into subnetworks is illustrated in figure (4.1). Note that this arrangement is purely schematical and hence unrelated to the true position of the stations on the earth's surface. Subnetwork 1 is directly connected to subnetwork 3 through their common stations 1, 2 and 6. Similarly subnetwork 1 is directly connected to subnetwork 4 through their common stations 5, 9 and 10, and subnetwork 2 with subnetwork 3 through stations 3, 4 and 7. These direct connections are sufficient for the establishment of the shape and size of the entire network from the individual shapes and sizes of the subnetworks. There also exists an additional, though superfluous, direct connection namely that of subnetwork 2 to subnetwork 4 through stations 8, 11 and 12. Obviously subnetwork 4 could contain three new stations instead of 8, 11 and 12, e.g. stations 13, 14, 15, thus allowing for the determination of the shape and size of a network of as much as 15 stations in addition to the determination of satellite estimable parameters with the same set of observations. Table (4.1) depicts the observational pattern to be followed, i.e. which subnetworks observe which satellite at each epoch. In this example, odd-numbered epochs can be chosen to constitute session 1 and even-numbered epochs to constitute session 2. Each session has nine epochs which are sufficient for the determination of the shape and size of each subnetwork when differential ranges are observed and only clock offsets are included in the model (c.f. model HG3 with only one clock offset parameter per station).

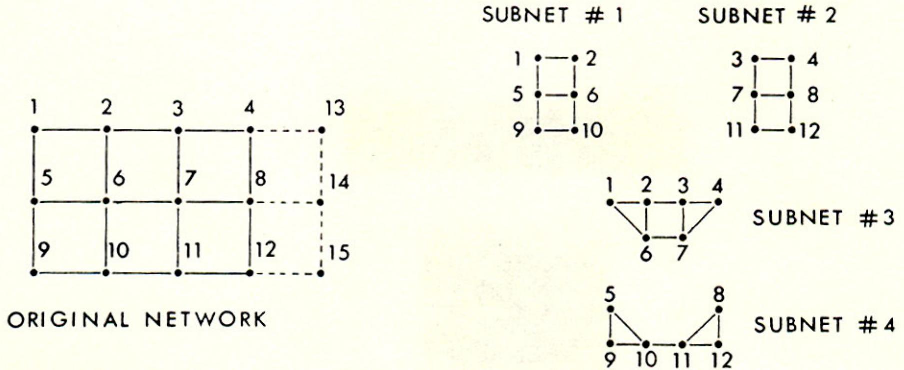

Figure 4·1

Table 4·1 - Observational Pattern

EPOCH #			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
SUBNETWORK	1	SATELLITE OBSERVED	1	–	2	–	3	–	4	–	5	–	6	–	1	–	2	–	3	–	SESSION # 1
	2		2	–	3	–	4	–	5	–	6	–	1	–	2	–	3	–	4	–	
	3		–	1	–	2	–	3	–	4	–	5	–	6	–	1	–	2	–	3	SESSION # 2
	4		–	3	–	4	–	5	–	6	–	1	–	2	–	3	–	4	–	5	

With regard to the satellite configuration in such an observational design, a careful choice of the satellites being observed may yield a sufficient number of satellite position vectors for the determination of their relative configuration. Then at each epoch, two ground station-to- satellite vectors are sufficient for the determination of the orientation of the network with respect to the satellite configuration. One such vector determines the orientation of the network up to a rotation about the direction of this vector, whereas the second such vector removes this degree of freedom.

4.2 GENERAL CRITERIA OF ESTIMABILITY FOR SUBNETWORK DESIGN

Consider the general case of a network with $g=k \cdot g_k$ stations subdivided into k subnetworks with equal number of stations $g_1, g_2, \ldots, g_k$ co-observing in each session. The choice of subnetworks with the same number of stations is not necessary but it leads to an easier determination of general criteria of estimability. For each observing session, the minimum number s_k of observation epochs necessary for the determination of the shape and size of each subnetwork can be obtained from Table (3.1) depending on the model involved (e.g. clock parameters solved for, etc.). As an example, we consider here the case where clock offsets only are included in the parameters to be estimated. The relevant inequality in this case is

$$s_k \geq \frac{4g_k - 7}{g_k - 4}$$

If n_k is the number of sessions necessary for the subnetworks to be directly connected, the total minimum number of necessary observation epochs is

$$s = n_k \cdot s_k \quad .$$

The number n_k of necessary sessions for the direct subnetwork connections can be determined as follows: except for the first session where we start with subnetworks 1 and 2, at each subsequent session subnetworks are introduced having three common stations with say subnetwork 1 and advancing by g_k-3 stations leading to a total of (n_k-1) (g_k-3) advancing stations. Since for a connection to be obtained this total number must be at least three, so that a direct connection takes place, we get

$$(n_k - 1)\ (g_k - 3) \geq 3$$

from which it follows that

$$n_k \geq \frac{g_k}{g_k - 3} \quad .$$

If superfluous connections are avoided, each new session after the first introduces $(g_k$-3) new stations so that the total maximum number of network stations that can be determined from the same number of observation epochs becomes

$$g_{max} = 2g_k + (n_k-1)(g_k-3) \quad .$$

TABLE 4.2. Estimability Conditions for a Network of g Stations Observing in Subnetworks with g_k Stations at n_k Observational Sessions

k	g	g_k	s_k	n_k	s	g_{max}
2	10	5	13	3	39	14
	12	6	9	2	18	15
	$2g_k$	7	7	2	14	$4g_k$-6

Table 4.2 summarizes these results for a network of g stations co-observing in two subnetworks of g_k stations at n_k observational sessions. Only clock offsets were considered in this case.

5. NUMERICAL ADJUSTMENT

Provided that the foregoing fundamental inequalities are satisfied a least squares adjustment of the collected differential observations can be carried out. In practice since we are primarily interested in the coordinates of the terrestrial points, a solution is often sought from a partitioned system of normal equations of the form

$$\begin{bmatrix} N_{cc} & N_{bc} \\ N_{bc}^{T} & N_{bb} \end{bmatrix} \begin{bmatrix} X_{c} \\ X_{b} \end{bmatrix} = \begin{bmatrix} A_{c}^{T} \\ A_{b}^{T} \end{bmatrix} P\ell \qquad (5.1)$$

where

$$A = \left[A_c \; A_b \right] \qquad (5.2)$$

is the first partitioning of the design matrix A of the network;

$$X = \begin{bmatrix} X_c \\ X_b \end{bmatrix} \qquad (5.3)$$

is the first partitioning of the unknown vector into a sub-vector X_c of coordinate unknowns (both terrestrial and satellite) and a sub-vector X_b of nuisance (non-geometrical and instrumental) parameters; N_{cc}, N_{bb} and N_{bc} are respectively the partitioned parts of the normal equations matrix pertaining to the partition of X.

A solution of equation (5.1) for X_c leads to

$$X_{c} = (N_{cc} - N_{bc} N_{bb}^{-1} N_{bc}^{T})^{-1} (A_{c}^{T} - N_{bc} N_{bb}^{-1} A_{b}^{T}) P\ell \; , \left[N_{bb}\right] \neq 0 \; . \qquad (5.4)$$

In a second partitioning scheme let X_c be further partitioned as

$$X_{c} = \begin{bmatrix} X_{g} \\ X_{s} \end{bmatrix} \qquad (5.5)$$

where X_g is the vector of unknown terrestrial coordinates and X_s is the vector of satellite coordinates (corrections to the satellite ephemerides). This second partitioning leads to a system of normal equations

$$\begin{bmatrix} N_{gg} & N_{gs} \\ N_{gs}^{T} & N_{ss} \end{bmatrix} \begin{bmatrix} X_{g} \\ X_{s} \end{bmatrix} = \begin{bmatrix} A_{g}^{T} \\ A_{s}^{T} \end{bmatrix} P\ell \quad \text{, with} \qquad (5.6)$$

$$\begin{bmatrix} N_{gg} & N_{gs} \\ N_{gs}^{T} & N_{ss} \end{bmatrix} = (N_{cc} - N_{bc} N_{bb}^{-1} N_{bc}^{T}) \quad \text{, and} \qquad (5.7)$$

$$\begin{bmatrix} A_g^T \\ A_s^T \end{bmatrix} = A_c^T - N_{bc}\, N_{bb}^{-1}\, A_b^T \quad . \tag{5.8}$$

The solution of (5.6) leads to

$$X_g = (N_{gg} - N_{gs}\, N_{ss}^{-1}\, N_{gs}^T)^{-1} (A_g^T - N_{gs}\, N_{ss}^{-1}\, A_g^T)\, P\ell \ , \quad [N_{ss}] \neq 0 . \tag{5.9}$$

However the system of normal equations (5.9) cannot be inverted because of the datum rank defect already pointed out in the previous section. In practice the datum rank defect is often solved by fixing six carefully chosen coordinates among the network ground stations. This leads to a third partitioning of the vector X_g as

$$X_g = \begin{bmatrix} X_{g_1} \\ X_{g_2} \end{bmatrix} \tag{5.10}$$

where the sub-vector X_{g_2} contains the six fixed coordinates chosen. Finally this partitioning results into a system of normal equations

$$\begin{bmatrix} N_{g_1g_1} & N_{g_1g_2} \\ N^T_{g_1g_2} & N_{g_2g_2} \end{bmatrix} \begin{bmatrix} X_{g_1} \\ X_{g_2} \end{bmatrix} = \begin{bmatrix} A^T_{g_1} \\ A^T_{g_2} \end{bmatrix} P\ell, \quad [N_{g_2g_2}] \neq 0 \tag{5.11}$$

where

$$\begin{bmatrix} N_{g_1g_1} & N_{g_1g_2} \\ N^T_{g_1g_1} & N_{g_2g_2} \end{bmatrix} = N_{gg} - N_{gs}\, N_{ss}^{-1}\, N_{gs}^T \quad , \quad \text{and} \tag{5.12}$$

$$\begin{bmatrix} A^T_{g_1} \\ A^T_{g_2} \end{bmatrix} = (A_g^T - N_{gs}\, N_{ss}^{-1}\, A_s^T) \quad , \tag{5.13}$$

leading to the sought solution vector for the ground stations

$$X_{g_1} = (N_{g_1g_1} - N_{g_1g_2}\, N^{-1}_{g_2g_2}\, N^T_{g_1g_2})^{-1} (A^T_{g_1} - N_{g_1g_2}\, N^{-1}_{g_2g_2}\, A^T_{g_2})\, P\ell . \tag{5.14}$$

6. A-PRIORI INFORMATION IN GPS SATELLITE NETWORKS

So far we have judged the estimability of parameters present in the model of differential GPS observations only by examining the deterministic part of the linearized model of equation (3.2) or

(3.9). In essence, we have tacidly associated estimability with determinability, i.e. the possibility to determine the true values for the independent parameters included into the model, if the true values of the observables were known (or in other words if the observational errors were approaching zero). The numerical solution in section 5 for the free network adjustment of differential GPS observations may be understood as tacidly assuming that there is no a-priori information inherent in the approximate values for the unknown parameters. The premise for this assumption however, is somewhat incorrect from the statistical point of view: in practice one always knows to certain extent what the value of X should approximately be. Indeed one has to know this approximate value X° for the evaluation of the design matrix, e.g. the evaluation of the unit vectors $\vec{e}^{\,i}_{\alpha}$, $\vec{e}^{\,i}_{\beta}$ in equation (3.2). In linear approximation, the a-priori information inherent in the approximate values X° is used essentially by adding the weight matrix

$$P_X = C^{-1}_{X^\circ} = C^{-1}(X^\circ) \tag{6.1}$$

(where C_{X° denotes the a-priori covariance matrix of the parameters) to the usual normal equation matrix of least squares adjustment [Vaníček and Krakiwsky, 1982]. That is, for the case on hand, starting from the partitioned normal equations (5.1) of a free network of differential GPS observations, we obtain

$$\begin{bmatrix} N_{g_1g_1}+C^{-1}(X^\circ_{g_1}) & N_{g_1g_2} & N_{g_1s} & N_{g_1b} \\ N^T_{g_1g_2} & N_{g_2g_2}+C^{-1}(X^\circ_{g_2}) & N_{g_2s} & N_{g_2b} \\ N^T_{g_1s} & N^T_{g_2s} & N_{ss}+C^{-1}(X^\circ_s) & N_{sb} \\ N^T_{g_1b} & N^T_{g_2b} & N^T_{sb} & N_{bb}+C^{-1}(X^\circ_b) \end{bmatrix}$$

$$\cdot \begin{bmatrix} X_{g_1} \\ X_{g_2} \\ X_s \\ X_b \end{bmatrix} = \begin{bmatrix} A^T_{g_1} P\ell \\ A^T_{g_2} P\ell \\ A^T_s P\ell \\ A^T_b P\ell \end{bmatrix} \tag{6.2}$$

assuming no correlation between the approximate values for the different sets of unknown parameters.

In principle, all the approximate values $X^\circ_{g_1}$, $X^\circ_{g_2}$, X°_s, X°_b result from some previous estimation process. Thus if the corresponding covariance matrices were to be known, they could be used in (6.2).

Unfortunately this is not usually the case at least for the satellite coordinates. Thus one has to look for reasonable approximations for $C^{-1}(X_s^\circ)$. Some obvious choices can be the following:

i) a-priori information for a set of osculating Keplerian elements

Lets assume that the cartesian coordinates X_s° are computed from a set of broadcasted Keplerian elements

$K_1 = a$, semi-major axis of the orbit

$K_2 = e$, eccentricity

$K_3 = i$, inclination of the orbital plane

$K_4 = \Omega$, right ascension of ascending nobe

$K_5 = \omega$, argument of perigee

$K_6 = T_\circ$, time of perigee passage

which have a covariance matrix $C(K_i^\circ), i=1,2,\ldots,6$ expressed in a reference frame attached to the true equator and equinox corresponding to the reference epoch for the orbital elements. Using the usual law of covariance propagation, the covariance matrix $C(X_s^\circ)$ to be used in (6.1) is obtained from

$$C(X_s^\circ) \;=\; S\, C(K^\circ)\, S^T \tag{6.3}$$

where S is the Jacobian of transformation from Keplerian elements into Cartesian coordinates.

It should be pointed out that the introduction of $C(K^\circ)$ in the normal equations contains a-priori information related to the datum definition since the Keplerian elements define entirely the reference system as geocentric with axes oriented along the Earth's rotation axis and in the direction of the Greenwich meridian. Hence in the previously derived fundamental inequalities, the datum defect reduces to zero and the number of observations is increased by 3 for each observed satellite position for a total of $s(g-1)+3s$ (i.e. $s(g-1)$ as before for the observed differential ranges and 3s for the added "pseudo-observations" because of the augmentation of the normal equations matrix by $C_s^{-1}(X^\circ)$). For example, the condition for zero c-defect for the G3 model takes the form

$$s(g-1) + 3s \geq 3g + 3s \tag{6.4a}$$

or

$$s(g-1) \geq 3g \tag{6.4b}$$

which is the same as (3.5) as it should be.

ii) a-priori information in the reference frame attached to the satellite

A viable alternative to using a-priori information in the form of $C(K^\circ)$ is to use a covariance matric $C(q^\circ)$ expressed in a reference frame attached to the satellite with the axes pointing in the along-, across- and out-of-plane directions (see fig. 6.1).

Using the rotation matrix [Kaula, 1966]

$$R_{Xq}(\tau) = R_3(-\Omega(\tau)+GAST)\cdot R_1(-i)\cdot R_3[-\omega(\tau)] \tag{6.5}$$

which transform the orbital coordinate system q into the cartesian coordinate system, we get the covariance matrix $C(X_s^\circ)$ as

$$C(X_s^\circ) = R\cdot C(q^\circ)\, R^T \quad . \tag{6.6}$$

to be used in (6.2).

Here it should be noted that $C(q^\circ)$ contains a-priori information on the network translation, that is the datum defect is in this case is reduced by 3. Furthermore the introduction of this a-priori information on the adjustment is equivalent to introducing 3s additional "pseudo-observations" in the previously derived fundamental inequalities. For example, the condition for zero c-defect for the G3 model takes the form

$$s(g-1) + 3s \geq 3g + 3s - 3$$

or

$$s \geq 3 \quad , \quad (g > 1)$$

which always holds irrespective of the number of the observing stations.

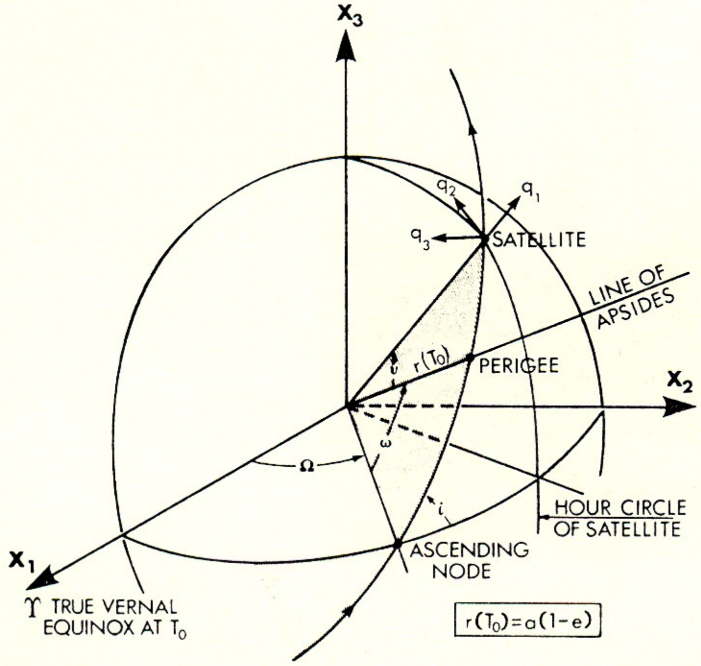

FIGURE 6.1

Keplerian Elements

7. EFFECT OF A-PRIORI CONSTRAINTS ON THE ADJUSTMENT RESULTS

In section 5, it was shown that for a free network adjustment based on differential GPS observations, the normal equations are developed for three sets of parameters: station coordinates, satellite coordinates and clock related parameters. Although in section 6, we have seen that the choice of a-priori information on the satellite coordinates generally reduces the d-defect in the free network adjustment, it is of interest to examine here the more general form of the normal equations to arrive at a proper understanding of what effect a-priori information on either the satellite parameters or the station coordinates of some of the stations has on the variance of the coordinates of the rest of the stations and the baseline components between such stations. This situation may naturally arise in network densification using differential range observations to GPS satellites.

Starting from the second partitioning scheme described in section 5, the least squares normal equations for station coordinates and satellite coordinates have the form given by equation (5.7). Assuming that a-priori information is now introduced for the satellite coordinates and for a set g_1 among the network ground stations, the system of normal equations takes the form

$$\begin{bmatrix} N_{g_1g_1} + P_1 & N_{g_1g_2} & N_{g_1s} \\ N^T_{g_1g_2} & N_{g_2g_2} & N_{g_2s} \\ N^T_{g_1s} & N^T_{g_2s} & N_{ss} + P_{ss} \end{bmatrix} \begin{bmatrix} X_{g_1} \\ X_{g_2} \\ X_s \end{bmatrix} = \begin{bmatrix} A^T_{g_1} & P\ell \\ A^T_{g_2} & P\ell \\ A^T_s & P\ell \end{bmatrix} \qquad (7.1)$$

where g_2 denotes here the subset of ground stations without a-priori information, P_1 is the weight matrix reflecting the a-priori knowledge in the coordinates of the station set g_1, and P_{ss} respectively is the weight matrix for the satellite coordinates defined according to section 6, i.e.

$$P_{ss} = C^{-1}(X^{\circ}_s) = \text{either} \left\{ \begin{array}{c} [S\ C(K^{\circ})\ S^T]^{-1} \\ \text{or} \\ [R\ C(q^{\circ})\ R^T]^{-1} \end{array} \right. \qquad (7.2)$$

After elimination of the satellite parameters, the covariance matrix for the station coordinates including a-priori information is

$$C_{X_g} = \begin{bmatrix} (N_{g_1g_1} + P_1) - N_{g_1s}(N_{ss}+P_{ss})^{-1}N^T_{g_1s} & N_{g_1g_2} - N_{g_1s}(N_{ss}+P_{ss})^{-1}N^T_{g_2s} \\ N^T_{g_1g_2} - N_{g_2s}(N_{ss}+P_{ss})^{-1}N^T_{g_1s} & N_{g_2g_2} - N_{g_2s}(N_{ss}+P_{ss})^{-1}N^T_{g_2s} \end{bmatrix}^{-1}$$

(7.3a)

$$= \begin{bmatrix} B_{11} & B_{12} \\ B_{12}^T & B_{22} \end{bmatrix}^{-1} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \tag{7.3b}$$

where the elements of the covariance matrix are given by

$$Q_{11} = [B_{11} - B_{12}\, B_{22}^{-1}\, B_{12}^T]^{-1} \tag{7.4a}$$

$$\begin{aligned} &= [(N_{g_1g_1} + P_1) - N_{g_1s}(N_{ss} + P_{ss})^{-1}N_{g_1s}^T \\ &- (N_{g_1g_2} - N_{g_1s}(N_{ss} + P_{ss})^{-1}N_{g_2s}^T) \\ &\cdot (N_{g_2g_2} - N_{g_2s}(N_{ss} + P_{ss})^{-1}N_{g_2s}^T)^{-1} \\ &\cdot (N_{g_1g_2}^T - N_{g_2s}(N_{ss} + P_{ss})^{-1}N_{g_1s}^T)]^{-1} \end{aligned} \tag{7.4b}$$

$$Q_{22} = [B_{22}^{-1} + B_{22}^{-1}\, B_{12}^T\, Q_{11}\, B_{12}\, B_{22}^{-1}] \tag{7.5a}$$

$$= [B_{22} - B_{12}^T\, B_{11}^{-1}\, B_{12}]^{-1} \tag{7.5b}$$

$$\begin{aligned} &= [(N_{g_2g_2} - N_{g_2s}\,(N_{ss} + P_{ss})^{-1}N_{g_2s}^T) \\ &- (N_{g_1g_2}^T - N_{g_2s}(N_{ss} + P_{ss})^{-1}N_{g_1s}^T) \\ &\cdot ((N_{g_1g_1} + P_1) - N_{g_1s}(N_{ss} + P_{ss})^{-1}N_{g_1s}^T)^{-1} \\ &\cdot (N_{g_1g_2} - N_{g_1s}(N_{ss} + P_{ss})^{-1}N_{g_2s}^T)]^{-1} \end{aligned} \tag{7.5c}$$

$$Q_{12} = -Q_{11}\, B_{12}\, B_{22}^{-1} \tag{7.6a}$$

$$\begin{aligned} &= -Q_{11}(N_{g_1g_2} - N_{g_1s}(N_{ss} + P_{ss})^{-1}N_{g_2s}^T) \\ &\cdot (N_{g_2g_2} - N_{g_2s}\,(N_{ss} + P_{ss})^{-1}N_{g_2s}^T)^{-1} \end{aligned} \tag{7.6b}$$

$$Q_{21} = Q_{12}^T\ . \tag{7.7}$$

Having determined the covariance for the station coordinates, the covariance matrix for the baseline components between stations in the g_1 and g_2 station sets respectively can be easily obtained by a linear transformation

$$C_{\Delta_{g_1 g_2}} = [-I \quad I] \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \begin{bmatrix} -I \\ I \end{bmatrix} \tag{7.8a}$$

$$= Q_{11} + Q_{22} - Q_{12} - Q_{12}^T \tag{7.8b}$$

$$= Q_{11} + Q_{22} + Q_{11} B_{12} B_{22}^{-1} + B_{22}^{-1} B_{12}^T Q_{11} \quad . \tag{7.8c}$$

The effect of a-priori information on the covariance matrices (7.4) through (7.8) can now be considered for the following cases:

i) satellite parameters constrained and no knowledge of the g_1 station set coordinates ($P_{ss} \to \infty$, $P_1 \to 0$)

Under these assumptions, equations (7.4) through to (7.7) reduce to

$$C_{X_{g_1}} = [N_{g_1 g_1} - N_{g_1 g_2} N_{g_2 g_2}^{-1} N_{g_1 g_2}^T]^{-1} \tag{7.9}$$

$$C_{X_{g_2}} = [N_{g_2 g_2} - N_{g_1 g_2}^T N_{g_1 g_1}^{-1} N_{g_1 g_2}]^{-1} \tag{7.10}$$

$$C_{\Delta_{g_1 g_2}} = C_{X_{g_1}} + C_{X_{g_2}} + C_{X_{g_1}} N_{g_1 g_2} N_{g_2 g_2}^{-1} + N_{g_2 g_2}^{-1} N_{g_1 g_2}^T C_{X_{g_1}} \tag{7.11}$$

ii) satellite parameters partially known and g_1 station set coordinates constrained ($0 < P_{ss} < \infty$, $P_1 \to \infty$)

$$C_{X_{g_1}} = 0 \tag{7.12}$$

$$C_{X_{g_2}} = [N_{g_2 g_2} - N_{g_2 s} (N_{ss} + P_{ss})^{-1} N_{g_2 s}^T]^{-1} \tag{7.13}$$

$$C_{\Delta_{g_1 g_2}} = [N_{g_2 g_2} - N_{g_2 s} (N_{ss} + P_{ss})^{-1} N_{g_2 s}^T]^{-1} = C_{X_{g_2}} \tag{7.14}$$

iii) satellite parameters constrained and g_1 station set coordinates constrained ($P_{ss} \to \infty$, $P_1 \to \infty$)

$$C_{X_{g_1}} = 0 \tag{7.15}$$

$$C_{X_{g_2}}^{-1} = N_{g_2 g_2}^{-1} \tag{7.16}$$

$$C_{\Delta_{g_1 g_2}} = N_{g_2 g_2}^{-1} \tag{7.17}$$

iv) satellite parameters and g_1 station set coordinates partially known ($0 < P_{ss} < \infty$, $0 < P_1 < \infty$)

$$C_{X_{g_1}} = \text{equation (7.4)}$$

$$C_{X_{g_2}} = \text{equation (7.5)}$$

$$C_{\Delta_{g_1 g_2}} = \text{equation (7.8)} .$$

A comparison of these covariance expressions lead to some useful conclusions on how the results may vary under different circumstances of usage of a-priori information. For instance, cases (i) and (iii) demonstrate that when the satellite orbit is assumed known, the uncertainty in the coordinates of stations in the g_2 set increases as the uncertainty of the coordinates of stations g_1 increases. A similar increase in the uncertainty of the coordinates of the stations in the g_2 set is to be expected when the coordinates of the stations in the g_1 set are constrained and the uncertainty of the satellite orbit increasses (case iii) vs case (ii)), and a further increase is to be expected as the uncertainty in the coordinates of the stations in the g_1 set also increases (case (ii) vs case (iv)).

For the baselines components, the inspection of the covariances (7.14) and (7.17) indicates that if the coordinates of the observing stations in the g_1 set are assumed known, increasing the uncertainty in the satellite orbit leads to an increased uncertainty in the components of baselines connecting g_1 and g_2 stations; also, if the satellite orbit is known, increasing the uncertainty of the station coordinates in the g_1 set also increases the uncertainty in the components of the g_1-g_2 baseline (c.f. equation (7.11) and (7.17)).

Finally it is interesting to examine these covariance expressions in the case of very short baselines. In this case, it is

$$N_{g_1 g_2} \simeq -N_{g_2 g_2} \tag{7.18a}$$

and

$$B_{12} \simeq -B_{22} . \tag{7.18b}$$

In case (i), it can easily be seen that even if the satellite orbit is known, the second term within the brackets in equation (7.9) and (7.10) are nearly equal to the first term thus leading to a covariance matrix for the coordinate components which tends to be singular as the baseline g_1-g_2 decreases. By contrast the last two terms in equation (7.11) are nearly identical (but with opposite sign) to the additive sum of the first two terms (i.e. $C_{\Delta_{g_1 g_2}} \to 0$) thus leading to a very small baseline component covariance matrix even in the presence of large satellite errors, a fact well which is known for applications of differential GPS operations but nevertheless intuitively pleasing to see in this connection.

8. SUMMARY AND CONCLUSIONS

We have considered the free network adjustment of differential range observations to GPS satellites. It was shown that an analysis of the configuration defects is indispensable in designing satellite networks based on differential GPS observations especially in view of a usually considerable complexity of the models involved. These models relate the values observed by means of GPS measuring techniques with geometric parameters (e.g. station positions) and dynamic ones (e.g. satellite motion parameters, earth rotation parameters and parameters of the gravitational field). In addition, an intermediary role of these models is fulfilled by various additional parameters describing for instance clock related behaviour, atmospheric delays of the signals, etc. In the present discussion, an analysis of the configuration effects was carried out by considering only station and satellite coordinate parameters as the fundamental parameters of the network; that is parameters of dynamic type like earth rotation, etc. have not been considered. Such an analysis will require a separate study of its own. Also in the adjustment, only clock related parameters were included in the model.

Having considered various adjustment models, it was shown that in order to avoid configuration deficiencies in free satellite GPS networks, certain inequalities will have to be fulfilled. When applied to simultaneous observing campaigns of subnetworks, these inequalities lead conveniently to optimal observational patterns from which useful receiver deployment strategies may be defined.

The defects of other types as the datum defect and the defect of ill-conditioning are strongly associated with the configuration defects. The d-defect results from our inability to define the reference system through these measurements. The i-defect may often be caused since the values of the observed quantities do not usually belong exactly to the "determinantal loci" [Tsimis, 1973], for which the network parameters are undefined, but they usually belong to the environment of these loci. As a result, one may arrive at near zero determinants of the normal equations which, besides causing numerical difficulties in solving such equations, influence the propagation of observational errors to the parameter values as well as lead to a high heterogeneity of solutions with respect to accuracy. To improve this situation, we proposed here the use of a-priori information inherent in the approximate values for the unknown parameters and pointed out the effects of a-priori information under different circumstances of usage. It is felt that particularly the indiscriminant use of impirical values for the a-priori information on the approximate coordinates for the satellite positions may lead to unsatisfactory results, in which case methods such as the variance-component-estimation [Grafarend et al., 1980] may be an alternative convenient tool to get more appropriate values directly from the available data set. This approach should also give correlations between subsequent satellite points within each pass as well as points between passes.

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